Credit Frictions and the Formal-Informal Employment Dynamics

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Abstract

According to the Brazilian labor force survey data, entry into formal employment over the business cycle is more volatile and procyclical than entry into informal employment. We argue that this difference is driven by formal and informal firms having different degrees of access to credit and exposure to credit market conditions. Specifically, formal firms are directly affected by varying over the business cycle access to credit, which strongly influences their hiring behavior. To demonstrate this mechanism, we build a two-sector dynamic stochastic general equilibrium model with search and matching frictions in labor markets and credit frictions in the financial market. The model generates a countercyclical risk premium on funds borrowed by firms in the formal sector, replicates moments of formal and informal employment observed in the data and explains several cyclical patterns of worker flows.
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1 Introduction

The issue of widespread informal employment or the underground economy in developing countries has attracted significant interest. According to Leandro and Friedrich (2019), the size of the informal economy (i.e., not recorded in official statistics) in low-income and emerging economies is estimated to be around 30% of GDP on average, with estimates for some countries exceeding 60% of GDP. Similarly, the share of workers employed in the informal sector is commonly estimated to be significant in many emerging and developing countries. Studies on informality have mainly concentrated on identifying the reasons why informal employment exists and what factors in the long-run determine their size and importance (see Boeri and Garibaldi (2005), Albrecht et al. (2009), Günther and Launov (2012) among others). Recent literature has also focus on the cyclical properties of informal employment (see Loayza and Rigolini (2006), Bosch and Maloney (2008), Bosch and Esteban-Pretel (2012) and Fiess et al. (2010), Gustavo and Carlos (2018), Horvath (2018), Colombo et al. (2019)). Following this latter stream of the literature, this paper analyzes the cyclical properties of worker flows between formal and informal employment and unemployment in a large developing country, Brazil. It also builds a dynamic stochastic general equilibrium model (DSGE) with search and matching and credit market frictions to explain observed empirical patterns.

Using the Brazilian labor force survey data, our analysis shows that the job finding rate from unemployment into formal employment is procyclical over the business cycle and more volatile than entry into informal employment. Direct transition from informal to formal employment is also relatively more procyclical and volatile than the transition from formal into informal jobs. Separations from informal employment exhibit countercyclical behavior. Similarly, the share of the labor force employed formally is procyclical, while the share of informal workers is countercyclical. The unemployment rate also exhibits strong countercyclical behavior. We suggest that these empirical observations, particularly the entry into formal employment, are driven by varying access to external credit over the business cycle by firms that operate formally. On the other hand, hiring in the informal sector is not directly affected by fluctuations in the credit conditions. Therefore, the dynamics of the job finding rate for informal jobs over the business cycle is different from the one observed for the formal sector employment. To study this mechanism, we build a dynamic stochastic general equilibrium (DSGE) model, in which both types of employment arrangements exist and formal firms have access to external credit.

We calibrate the model to data for Brazil and show that the model is consistent with the business cycle properties of different segments of the country's labor market. The model replicates the dynamics of labor market flows between formal and informal employment and unemployment. In the model, financial conditions that vary over the business cycle have a disproportionately stronger impact on the operation of formal firms and the business cycle properties of the two labor market segments (i.e., formal and informal) are generated through their different degree of exposure to credit market conditions. While negative technology shocks affect the model economy by reducing the total output and employment, the impact on each segment of the labor market (particularly on the hiring margin) is different.

This paper is closely related to several theoretical and empirical studies in the literature. Bosch and Maloney (2008) analyze worker flows between formal and informal employment using Brazilian and Mexican labor force surveys. The study finds that the share of formal employment in these countries is driven by highly procyclical job finding rates (for formal jobs) and the difficulty of finding formal jobs during downturns. In a related study, Hoek (2007) argues that differences in finding and separation rates between formal and informal jobs over the business cycle in Brazil are driven by the existence of high dismissal costs that induce formal firms to adjust on the hiring margin in recessions. Bosch and Esteban-Pretel (2012), on the other hand, propose a model where differences in the behavior of worker flows into formal and informal jobs are due to differences in the realization of match-specific productivity shocks, where higher productivity matches between employers and workers are more likely to result in a formal job contract.

Our paper is closest to the following studies including Fernández and Meza (2015), Horvath (2018), Colombo et al. (2019) and Leyva and Urrutia (2020). Fernández and Meza (2015) study the cyclical properties of aggregate employment in Mexico and show how an otherwise standard business cycle model of a small open economy but with explicitly modeled informal employment can explain the differences in business cycle properties of aggregate employment between developed and emerging economies. Horvath (2018) builds a two-sector business cycle model of a small open economy and shows how the presence of an imperfectly measured informal sector in the economy explains the relatively higher volatility of the ratio of consumption to output observed in emerging economies. Colombo et al. (2019) analyze the impact of financial crises on labor relocation between formal and informal employment. Similar to our paper, along with modeling formal and informal segments of the labor market, they incorporate the banking frictions into their model. In their model, the banking crises lead to a decline in the rate of creation of formal jobs and the informal employment plays the role of a buffer for workers during such episodes. In contrast to these studies, in this paper, we explicitly model the interaction between credit frictions and both segments of the labor market. Informal employment in our model is also assumed to be subject to search-and-matching frictions and it is not modeled as self-employment (as it's commonly done in the relevant literature). The main contribution of the paper is demonstrating the importance of financial frictions in generating the varying response of two segments of the labor market to negative productivity shocks.

The paper proceeds as follows. In the next section, we review key empirical observations on the dynamics of formal and informal employment and the evolution of job finding and separation rates over the business cycle in Brazil. After laying out the key features of the model, we discuss several simulation results and the model's ability to explain the empirical observations. Section 5 concludes.

2 Empirical observations

We use data from the Brazilian Monthly Employment Survey (*Pesquisa Mensal de Emprego*, PME) to examine the business cycle properties of transitions between formal and informal employment and unemployment. PME is one of few labor market surveys in developing countries that allows us to study labor market dynamics at a relatively high monthly frequency. It is a rotating panel survey of households in major urban areas in Brazil conducted by the Brazilian Institute for Geography and Statistics between 1980 and 2016 (*Instituto Brasileiro de Geografia e Statistica*, IBGE).[1] We use the data starting from March 2002 until March 2011. PME had been administered since early 1980s but we use data only after 2002, as the survey has been drastically changed in 2002 and definitions of certain labor market states are not consistently comparable across old and new survey designs. In this respect, our results complement the findings of Bosch and Esteban-Pretel (2012), who conduct the same analysis using the PME data from 1982 until the first quarter of 2002.

Determining the size of informal economy and the share of informal employment in the labor force is not a straightforward task. The PME survey allows us to determine the size of informal employment using several criteria. In the survey, tt is possible to identify if an individual holds a so-called "work card" (*"carteira de trabalho assinada"*), which is issued to every formally employed worker. The survey also provides information on the size of a firm, where a worker is employed, and whether mandatory social security payments are made or not. Using this criterium, we define informal workers as those employed individuals, who do not hold the work card. The sample is limited to workers of age between 15 and 65 and individuals out of the labor force are excluded.

To calculate the moments that characterize the cyclical behavior of job finding and separation rates, we use the panel structure of PME and calculate the gross flows of workers between three labor market states across two months. Figures 2a-2c show the evolution of job finding, separation and transition rates. To estimate the moments of these indicators, we smooth obtained monthly series using a moving average filter, take quarterly averages and detrend the quarterly series using the Hodrick-Prescott filter with the smoothing parameter of $\lambda =$ 1600. Then, we compute (i) the correlations of labor market variables with smoothed and detrended quarterly GDP series and (ii) the volatility of labor market variables relative to the volatility of quarterly GDP series.[2] Results are presented in Table 1.[3]

[1] In this survey a household is followed for four consecutive months before it is dropped from the sample for eight months, after which the household is again introduced into the sample for four months. By the survey design, a quarter of the households leaves the sample every month and they are substituted for by a new set of households.

[2] Quarterly GDP series (at 1995 constant prices) is provided by the IBGE.

[3] We also repeated the analysis using a different definition of informal employment, where following Bosch and Esteban-Pretel (2012) we added self-employed workers (not identified as professionals or technicians) to the pool of informal salaried employees. Results are presented in Table 8 in Appendix A and they are very similar to those presented in Table 1.

Table 1: Correlation and relative volatility of labor market variables in Brazil.

x	ϕ^F	ϕ^I	λ^F	λ^I	τ^{FI}	τ^{IF}	u	n^F
$\rho(y,x)$	0.49	-0.23	-0.13	-0.41	-0.17	0.18	-0.50	0.28
σ_x/σ_y	4.45	2.94	4.52	5.16	1.43	2.24	2.87	0.58

Notes: Superscripts F and I indicate formal and informal employment, respectively; y refers to real quarterly GDP, x refers to the other variables: ϕ denotes the job finding rate, λ is the job separation rate, τ is the direct transition rate between formal and informal employment, u is the unemployment rate and n^F is the share of formal employment in total employment. Statistics are computed after taking natural logarithm and detrending original series using the HP filter with the smoothing parameter $\lambda = 1600$.

The job finding rate from unemployment into formal employment is procyclical, while the job finding rate into informal employment is somewhat countercyclical. Separation rates are both countercyclical but the rate is more countercyclical and volatile from informal employment. Similarly, direct transitions from informal to formal jobs are procyclical and more volatile. We find that the direct transition to informal employment from formal jobs is acyclical. The unemployment rate is countercyclical and about three times more volatile than the output series. On the contrary, the share of formal employment in total employment is somewhat procyclical and about half as volatile as the output. These estimates are overall consistent with those of Bosch and Esteban-Pretel (2012) obtained using the PME data for the earlier period (before 2002). The key difference between the two estimates is that the job finding rate into informal employment in the period after 2002 appears more volatile and countercyclical. In addition, according to our estimates, the relative volatility of the unemployment rate is about twice smaller than the estimate for the period before 2002. In the literature, both pro- or countercyclicality of informal employment (and the corresponding job finding rate) over the business cycle have been observed, and the difference across the two periods is not surprising.

We argue that this differential behavior of entry into formal and informal employment is driven by changes in the credit conditions over the business cycle directly affecting formal firms. Unlike formal firms, informal firms are commonly not able to use the full value of their assets as collateral and they lack credit history, which limits their ability to obtain formal credit. Empirical evidence also suggests that higher levels of formality in firms' operations are associated with higher access to formal credit. This can also be seen in the macro data. Figure 1 in Appendix A plots the ratio of domestic credit to private sector (as share of GDP) against an estimated size of the informal economy. A similar negative relationship between informality and formal credit has also been established using firm-level data. For instance, Dabla-Norris and Koeda (2008) use the firm-level data from several transition economies and find that a relatively higher level of informality in operations does reduce access to formal bank credit. In addition, they establish only one-way causality – informality leads to difficulties accessing credit and not vice versa. Based on firm-level data for a large number of countries, Gatti and Honorati (2008) also find evidence for a relationship between the degree of formality within firms and their ability to access formal credit.[4]

As for evidence on the use of credit and informality in Brazil, we refer to results of the Survey of Urban Informal Economy (*Pesquisa de Economia Informal Urbana*) conducted by the IBGE in 2003. The survey collected information on self-employed workers and small firms with at most five workers. Due to its design, this survey provides information pertinent only to a limited share of workers surveyed in the PME. Nevertheless, it still provides useful information on the use of external finance in small informal firms and by self-employed workers. According to the survey results, about 94% of informal firms and self-employed workers used no external finance in their everyday operations within the three months prior to the survey (IBGE, 2005). Thus, to reflect this observation, we assume in the model that only formal firms have access to external credit.

3 Model

3.1 Economic environment

The model is in the class of real business cycle models augmented with a labor market subject to search and matching frictions. Such a model is extended to accommodate two production sectors, formal and informal, and the financial sector. Labor markets in both production sectors are assumed to be subject to search and matching frictions. The perfectly competitive financial sector has access to an unlimited source of funds and it lends them to firms in the formal sector. Agents in this economy are (i) households with household members employed in both production sectors, (ii) firms that use labor as the only factor of production, and (iii) an infinite number of lenders in the financial sector. In the following subsections, we discuss each of these sectors and agents in more detail.

3.2 Labor market

Labor is homogeneous in terms of its productivity. This way of modeling formal and informal labor markets is similar to the recent literature that abstracts from productivity differences among workers.[5] In other words, we assume that the workers are homogeneous and are not directed to a specific sector to look for a job. To simplify the analysis and to keep the model tractable, we assume that all transitions between formal and informal employment occur through unemployment, i.e., direct transitions between formal and informal jobs are not allowed. This simplifies the model and allows us to focus on the interaction of the financial sector with the real side of the model economy.

[4]See also relevant theoretical studies on the relationship between formality, contract enforcement and access to credit, e.g., Straub (2005) and Quintin (2008).

[5]See Charlot et al. (2010) and Ulyssea (2010).

Labor markets in both sectors are subject to search and matching frictions. Matching frictions are characterized by the standard Cobb-Douglas function, which maps unemployed household members and vacancies in both sectors to matches:

$$M_t^k = \psi^k u_t^\alpha (\upsilon_t^k)^{1-\alpha}, \quad k = I, F, \tag{1}$$

where superscripts I and F indicate informal and formal sectors, respectively. The term ψ^k governs the efficiency of the matching technology in both sectors, u_t denotes the unemployment rate and υ_t^k denotes vacancies created in period t by firms in sector k. As in the closely related literature (Ulyssea, 2010; Charlot et al., 2010) we assume that the matching process is less frictional in the informal sector, i.e. $\psi^F < \psi^I$. This assumption can also be interpreted as capturing the observation that, on average, it is easier to fill vacancies in the informal sector. In both sectors, firms find a worker at a rate M_t^k/υ_t^k, while unemployed workers find employment in either of the sectors at rate M_t^k/u_t.

Let's define the labor market tightness in both sectors as $\theta_t^k \equiv \upsilon_t^k/u_t$. Then, using the matching function (1) we can define the rate, at which firms fill their vacancies, as $M_t^k/\upsilon_t^k \equiv m_t^k = \psi^k(\theta_t^k)^{-\alpha}$ and the job finding rate as $M_t^k/u_t = \theta^k m_t^k$. Every period in each sector an exogenous fraction δ^k of employed workers lose their job. As is shown in Table 7, separation rates differ across types of employment and, therefore, we assume that $\delta^F < \delta^I$, which implies that the job turnover is higher among workers that are employed informally. Employment in each sector evolves according to the following law of motion

$$n_t^k = (1-\delta^k)n_{t-1}^k + m_{t-1}^k \upsilon_{t-1}^k. \tag{2}$$

Workers are allowed to be in three states – employed in either the formal or informal sector or unemployed. Thus, we define the unemployment rate as $u_t = 1 - \sum_k n_t^k$. Using this definition we can formulate the law of motion for unemployment in this economy as

$$u_t = u_{t-1} + \delta^F n_{t-1}^F + \delta^I n_{t-1}^I - \sum_k m_{t-1}^k \upsilon_{t-1}^k. \tag{3}$$

3.3 Households

There is a representative household in this economy consisting of a continuum of measure one of family members. We adopt the "large family" framework of Merz (1995) and Andolfatto (1996), which ensures that all household members can enjoy the same level of consumption regardless of labor market status of its members. Before we formulate the intertemporal problem that a household solves every period, we define the intra-temporal problem of allocating consumption between two consumption goods produced in each sector. Specifically, a household consumes good c_t, which is a composite of goods produced in formal and informal sectors, c_t^F and c_t^I, and it is defined as $c_t = (\varkappa(c_t^F)^\rho + (1-\varkappa)(c_t^I)^\rho)^{1/\rho}$. The parameter $0 < \varkappa < 1$ denotes the share of goods produced in the formal sector and ρ is the elasticity of substitution between c_t^F and c_t^I. The household solves the problem of minimizing the total costs of purchasing one unit of the composite consumption good. The price of the composite consumption good c_t is fixed to one throughout the model. Relative prices of goods produced by formal and informal firms are p_t^F and p_t^I, respectively. The solution to this intra-temporal problem yields the following expressions that govern the allocation of consumption across formal and informal goods

$$p_t^F = \varkappa \left(\frac{c_t^F}{c_t} \right)^{\rho-1} \tag{4}$$

$$p_t^I = (1-\varkappa) \left(\frac{c_t^I}{c_t} \right)^{\rho-1}. \tag{5}$$

Employed household members supply labor to firms in both sectors and collect wages. The unemployed receive fixed unemployment benefits, and all household members collect returns on state-contingent one-period risk-free bonds. Total household income is used for consumption, payment of lump-sum taxes, and purchase of one-period risk-free bonds. Let's denote the value of a household at time t as $\mathcal{H}_t$. Thus, a household solves the following problem

$$\mathcal{H}_t = \max_{c_t, b_t} \left\{ \frac{c_t^{1-\frac{1}{\xi}} - 1}{1-\frac{1}{\xi}} + \beta \mathbb{E}_t \left\{ \mathcal{H}_{t+1} \right\} \right\}$$

subject to

$$w_t^F n_t^F + w_t^I n_t^I + u_t \kappa + (1+r_{t-1}) b_{t-1} + \pi_t = c_t + \tau + b_t, \tag{6}$$

where β is a discount factor, w_t^F and w_t^I denote wages, κ is fixed unemployment benefit, b_t denotes real bonds, τ is fixed lump-sum tax payment and π_t is dividend payment due to ownership of firms. Solving this problem yields the following first order conditions

$$\lambda_t^c = c_t^{-\frac{1}{\xi}} \tag{7}$$

$$\lambda_t^c = \beta(1+r_t)\mathbb{E}_t \lambda_{t+1}^c, \tag{8}$$

where λ_t^c is a Lagrange multiplier associated with the household's budget constraint. Let's define the probability of finding a job in one of the sectors as $\phi_t^k = \psi^k (\theta_t^k)^{1-\alpha}, \ k = F, I$. Then, we can reformulate the laws of motion for employment for a household member in each sector as

$$n_{t+1}^F = (1-\delta^F) n_t^F + \phi_t^F u_t \tag{9}$$

$$n_{t+1}^I = (1-\delta^I) n_t^I + \phi_t^I u_t, \tag{10}$$

Using these reformulated laws of motion we determine the following value functions, which define the marginal value of being in each labor market state,

$$\mathcal{H}_{n,t}^F = \lambda_t^c w_t^F + \beta \mathbb{E}_t \left\{ (1-\delta^F) \mathcal{H}_{n,t+1}^F + \delta^F \mathcal{H}_{u,t+1} \right\} \tag{11}$$

$$\mathcal{H}_{n,t}^I = \lambda_t^c w_t^I + \beta \mathbb{E}_t \left\{ (1-\delta^I) \mathcal{H}_{n,t+1}^I + \delta^I \mathcal{H}_{u,t+1} \right\} \tag{12}$$

$$\mathcal{H}_{u,t} = \lambda_t^c \kappa + \beta \mathbb{E}_t \left\{ (1-\phi_t^F - \phi_t^I) \mathcal{H}_{u,t+1} + \phi_t^F \mathcal{H}_{n,t+1}^F + \phi_t^I \mathcal{H}_{n,t+1}^I \right\}. \tag{13}$$

Terms $\mathcal{H}^F_{n,t}$, $\mathcal{H}^I_{n,t}$ and $\mathcal{H}_{u,t}$ denote the value of formal and informal employment and unemployment, respectively. Tax revenues are used by the government to finance the payment of unemployment benefits and the government budget is always balanced.

3.4 Financial sector and the firm-lender relationship

The relationship between formal firms and lenders is modeled as in Petrosky-Nadeau (2011) and Chugh (2009), which model the relationship following Carlstrom and Fuerst (1997). In particular, the relationship between lenders and borrowers is assumed to be subject to frictions and costly state verification. The advantage of Petrosky-Nadeau (2011) and Chugh (2009) for our model is that the effect of the financial sector on a firm can be modeled without modeling the process of capital accumulation and investment. In the model, changes in the costs of financing directly affect the hiring decision of firms.

There is a perfectly competitive financial sector, consisting of an infinite number of lenders. These lenders have unlimited access to funds that can be lent only to formal firms. We assume that vacancy creation costs and wages in the formal sector should be paid out before the production in the formal sector occurs. To finance their operations, firms in the formal sector use their net worth. We also assume that the net worth is not sufficient every period, and formal firms should resort to external funds to finance their operational needs. One interpretation of this assumption is that these external funds help a firm gain and maintain access to more productive production technology. The net worth of a formal firm, nw_t, is defined below.

Let's define formal firm's period t operational costs as $\mathcal{C}^F_{it}$ (firms are indexed by i). As we have specified above, a formal firm always needs to resort to external finance to be able to operate, i.e., $\mathcal{C}^F_{it} > nw_{it}$. This implies that the size of the loan is defined by $\mathcal{C}^F_{it} - nw_{it}$. To introduce frictions into the borrower-lender relationship, we assume that in addition to aggregate productivity shock, formal firms are subject to an idiosyncratic productivity shock, ω_{it}, with mean $\mathbb{E}(\omega_{it}) = 1$. We denote by $\Phi(\omega)$ cumulative distribution function and by $\phi(\omega)$ density function of this shock. Idiosyncratic productivity shock is identically and independently distributed across time and formal firms. The realization of this shock is revealed after the debt contract between a formal firm and a lender is signed. The value of the shock is known only to a formal firm. The idiosyncratic shock introduces uncertainty into the firm-lender relationship and implies that some firms in the formal sector turn out to be more productive and successful than others. This feature of the model can be justified by the empirical observation that the share of firms operating formally is relatively higher in technically sophisticated sectors, they operate at a higher capital-labor ratio and they are key drivers of innovation and productivity growth in the economy. These features of the formal sector also imply that there is a relatively higher degree of uncertainty in the production process, which stems from introducing and making investments into new technologies. The idiosyncratic shock specifically captures this observation.

Let's define the revenues of a formal firm in terms of a composite consumption good as $\omega_{it} a_t n_{it}^F p_t^F$. Formal firms and lenders sign a "standard debt contract", which determines the cutoff level of idiosyncratic productivity, $\overline{\omega}_{it}$, and the size of the loan a firm takes in period t. This debt contract specifies that if the realization of an idiosyncratic productivity shock is below the threshold level, $\omega_{it} < \overline{\omega}_{it}$, the firm hands over to the lender all the revenues it collects. In case $\omega_{it} \geq \overline{\omega}_{it}$ the firm is able to pay back the loan and it keeps the rest of its revenues in the amount of $(\omega_{it} - \overline{\omega}_{it}) a_t n_{it}^F p_t^F$. This intra-period loan contract is paid off before agents move to the next period. A lender monitors firms that declare bankruptcy at a cost to exclude underreporting of revenues and bankruptcy by successful firms (to keep a larger share of revenues). This monitoring cost, denoted by μ_t, is a share of revenues that a lender receives after a firm declares bankruptcy. Following Petrosky-Nadeau (2011) we assume that monitoring cost is a function of aggregate productivity and it behaves countercyclically, i.e. $\mu_t = h(a_t),\ h'(a_t) < 0$ and $h''(a_t) < 0$.

Let's define the probability of being monitored by a lender after observing $\omega_{it} < \overline{\omega}_{it}$ as $G(\overline{\omega}_{it}) = \int_0^{\overline{\omega}_{it}} \omega_{it} d\Phi(\omega)$. Let's also define the lender's share of output before the monitoring costs are paid as $\Gamma(\overline{\omega}_{it}) = \overline{\omega}_{it}(1 - \Phi(\overline{\omega}_{it})) + G(\overline{\omega}_{it})$. A representative lender operates in a perfectly competitive financial sector and makes zero profit from lending to formal firms. Thus, a lender operates as long as the share of output it collects equals the sum of loans it gives out. This condition then allows us to define the lender's participation constraint as

$$(\Gamma(\overline{\omega}_{it}) - \mu_t G(\overline{\omega}_{it})) a_t p_t^F n_{it}^F = \mathcal{C}_{it}^F - nw_{it}, \tag{14}$$

where $\mathcal{C}_{it}^F - nw_{it}$ denotes the size of the loan to a formal firm and $\Gamma(\overline{\omega}_{it}) - \mu_t G(\overline{\omega}_{it})$ is the lender's share of firm's output net of monitoring costs. Lenders operate in a perfectly competitive financial sector. Therefore, in equilibrium, the terms of the debt contract maximize the expected profit of a formal firm. This simplified setup allows us to embed the process of determining the terms of the debt contract into a formal firm's profit maximization problem.

Restricting access to credit only to firms in the formal sector implies that informal firms are not strongly affected by changes in the cost of credit and financial market conditions. A related implicit assumption is that firms that operate formally and informally are not highly dependent on each other.[6] Available empirical evidence supports this assumption. Using the 2003 wave of the Brazilian survey of informal firms, Paula and Scheinkman (2010) check for the chain effect of formality and informality among firms. The study shows (empirically and theoretically) that the way how the value-added tax is collected and the tax credit is estimated induces formal firms to do business primarily with other formal firms. Equivalently, the same mechanism induces informal firms to do more business with other informal firms. Therefore, due to this evidence that formal and informal firms are not closely integrated through the production chain, the effect of varying cost of credit is most likely strongest within the group of formal firms.

[6]Though, as empirical evidence suggests labor flows between the two types of employment are not highly restricted.

3.5 Firms

Continuum of firms of size one in formal and informal sectors hire labor from households and produce according to the following production functions

$$y_{it}^F = a_t n_{it}^F \tag{15}$$
$$y_{it}^I = \gamma a_t n_{it}^I. \tag{16}$$

Firms in both sectors are subject to a common aggregate productivity shock a_t, but we further assume that production technology in the informal sector is strictly inferior to the formal one. We model this assumption by augmenting the production function of informal firms with an exogenous multiplicative parameter $0 < \gamma < 1$.[7] This assumption implies that for a given value of aggregate productivity, workers employed by formal firms engage in a more productive production process, while workers employed by informal firms are a factor in the less productive technology. As it is customary in the literature, the aggregate productivity is driven by the following AR(1) process and it is the only driving force in the model economy: $\log a_t = \varrho \log a_{t-1} + \varepsilon_t,\ 0 \leq \varrho < 1,\ \varepsilon \sim N(0, \sigma_\varepsilon^2)$. Value of the aggregate productivity shock is equal across formal and informal sectors, but the overall effect of the aggregate productivity shock across sectors would differ due to the existence of credit frictions.

Firms create vacancies every period, announce them to a common pool of the unemployed; after successful matches are formed, workers are employed next period. Vacancies in the informal sector can be opened at a lower cost than in the formal sector. On the contrary, formal sector firms incur non-negligible costs every time they decide to open a new formal vacancy. This assumption captures the fact that a formal vacancy should comply with labor, safety and other relevant regulation, which would incur higher costs to a firm.

Let's denote by $\mathcal{V}_{it}^F$ the value of a formal firm at time t. A firm solves the following problem

$$\mathcal{V}_{it}^F = \max_{\upsilon_{it}^F, \overline{\omega}_{it}} \left\{ (1 - \Gamma(\overline{\omega}_{it})) a_t p_t^F n_{it}^F + \beta \mathbb{E} \left\{ \Lambda_{t,t+1} \mathcal{V}_{it+1}^F \right\} \right\} \tag{17}$$

subject to

$$\mathcal{C}_{it}^F - n w_{it} = (\Gamma(\overline{\omega}_{it}) - \mu_t G(\overline{\omega}_{it})) a_t p_t^F n_{it}^F \tag{18}$$
$$n_{it+1}^F = (1 - \delta^F) n_{it}^F + m_t^F \upsilon_{it}^F, \tag{19}$$

where υ_{it}^F is the number of vacancies created by a formal firm, φ^F is a fixed cost of creating one vacancy and $\mathcal{C}_{it}^F = w_t^F n_{it}^F + \varphi^F \upsilon_{it}^F$ is the total operational cost of a firm. The term $1 - \Gamma(\overline{\omega}_{it})$ denotes the firm's share of output after necessary payments to the lender are made. According to this problem, a firm maximizes its value by choosing the number of vacancies to open and by determining the threshold level of the idiosyncratic productivity

[7] There is ample evidence that firms that operate informally are significantly less productive than formal firms.

shock specified in the debt contract. This maximization is performed subject to the lender's participation constraint (18) and the law of motion of employment in the formal sector (19). The solution to this problem yields the following first-order conditions

$$\lambda_t^\omega = \frac{\Gamma'(\overline{\omega}_{it})}{\Gamma'(\overline{\omega}_{it}) - \mu_t G'(\overline{\omega}_{it})} \tag{20}$$

$$\frac{\lambda_t^\omega \varphi^F}{m_t^F} = \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \mathcal{V}_{n,it+1}^F \right\}, \tag{21}$$

where λ_t^ω is a Lagrange multiplier associated with the lender's participation constraint and $\Lambda_{t,t+1} = \lambda_{t+1}^c / \lambda_t^c$ is a stochastic discount factor. λ_t^ω can also be interpreted as the shadow price of external funds. Expression (20) governs the evolution of $\overline{\omega}_t$ in equilibrium and (21) is formal firm's job creation condition. Taking the derivative of (17) with respect to n_{it}^F gives the following envelope condition

$$\mathcal{V}_{n,it}^F = (1 - \Gamma(\overline{\omega}_{it})) a_t p_t^F + \lambda_t^\omega \left((\Gamma(\overline{\omega}_{it}) - \mu_t G(\overline{\omega}_{it}))\, a_t p_t^F - w_t^F \right) + (1 - \delta^F) \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \mathcal{V}_{n,it+1}^F \right\}. \tag{22}$$

Then, using the first order condition (21) and the envelope condition (22) we reformulate the job creation condition of a formal firm and obtain

$$\frac{\lambda_t^\omega \varphi^F}{m_t^F} = \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \Omega(\overline{\omega}_{it+1}) a_{t+1} p_{t+1}^F - \lambda_{t+1}^\omega w_{t+1}^F + (1 - \delta^F) \frac{\lambda_{t+1}^\omega \varphi^F}{m_{t+1}^F} \right\} \right\}, \tag{23}$$

where $\Omega(\overline{\omega}_{it+1}) = 1 - \Gamma(\overline{\omega}_{it+1}) + \lambda_{t+1}^\omega (\Gamma(\overline{\omega}_{it+1}) - \mu_{t+1} G(\overline{\omega}_{it+1}))$.

Informal firms in the economy solve the following simpler problem

$$\mathcal{V}_{it}^I = \max_{v_{it}^I} \left\{ \gamma a_t p_t^I n_{it}^I - \mathcal{C}_{it}^I + \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \mathcal{V}_{it+1}^I \right\} \right\} \tag{24}$$

subject to

$$n_{it+1}^I = (1 - \delta^I) n_{it}^I + m_t^I v_{it}^I, \tag{25}$$

where $\mathcal{C}_{it}^I = w_t^I n_{it}^I + \varphi^I v_{it}^I$ is the total operational cost of an informal firm and φ_t^I is the fixed cost of creating one informal vacancy. This problem states that an informal firm maximizes the current profit and the discounted future value of the firm by choosing the number of vacancies to create in the current period subject to a law of motion of the labor force employed in the informal sector. Solving this problem yields the job creation condition in the informal sector

$$\frac{\varphi^I}{m_t^I} = \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \mathcal{V}_{n,it+1}^I \right\}, \tag{26}$$

which using the corresponding envelope condition can be reformulated to

$$\frac{\varphi^I}{m_t^I} = \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \gamma a_{t+1} p_{t+1}^I - w_{t+1}^I + (1 - \delta^I) \frac{\varphi^I}{m_{t+1}^I} \right\} \right\}. \tag{27}$$

Let's define the evolution of the formal firm's net worth. Net worth is the share of revenues a formal firm keeps after paying off the loan and it is defined as

$$nw_{it+1} = \zeta(1 - \Gamma(\overline{\omega}_{it}))a_t p_t^F n_{it}^F, \tag{28}$$

where $0 < \zeta < 1$ is necessary to rule out a situation, when a formal firm is able to self-finance its operational expenses. Using the lender's participation constraint (14), we can reformulate the expression for the evolution of the net worth as

$$nw_{it+1} = \zeta \left(a_t p_t^F n_{it}^F - (\mathcal{C}_{it}^F - nw_{it}) \left(1 + \frac{\mu_t G(\overline{\omega}_{it}) a_t p_t^F n_{it}^F}{\mathcal{C}_{it}^F - nw_{it}} \right) \right), \tag{29}$$

which explicitly shows that the net worth is the difference between formal firm's revenues, costs and debt related payments. The fraction $(\mu_t G(\overline{\omega}_{it}) a_t p_t^F n_{it}^F)/(\mathcal{C}_{it}^F - nw_{it})$ is equivalent to a premium on external funds that a formal firm borrows every period.

3.6 Wage bargaining

Wages in both sectors are determined through a standard Nash bargaining mechanism. Let's define the total surplus that formal firms and new matched workers share through the bargaining process as $\mathcal{S}_t^F = \mathcal{W}_t^F + \mathcal{V}_{n,t}^F$. The term $\mathcal{W}_t^F = \frac{\mathcal{H}_{n,t}^F - \mathcal{H}_{u,t}}{\lambda_t^c}$ determines the net value of being formally employed for a household member, where we divide the term $\mathcal{H}_{n,t}^F - \mathcal{H}_{u,t}$ by λ_t^c to represent the net value in terms of a composite consumption good. Using the expressions for the marginal value of being in the three labor market states (11)-(13) and the envelope condition for formal employment, we determine the total surplus that formal firms and matched workers share as

$$\begin{aligned} \mathcal{S}_t^F = {} & \Omega(\overline{\omega}_t) a_t p_t^F + (1 - \lambda_t^\omega) w_t^F - \kappa \\ & + (1 - \delta^F)\beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \mathcal{V}_{n,t+1}^F + \mathcal{W}_{t+1}^F \right\} \right\} \\ & - \beta \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left\{ \phi_t^F \mathcal{W}_{t+1}^F + \phi_t^I \mathcal{W}_{t+1}^I \right\} \right\}. \end{aligned} \tag{30}$$

Formal firms and the newly matched workers solve the following bargaining problem

$$\max_{w_t^F} \left(\mathcal{W}_t^F \right)^\eta \left(\mathcal{V}_{n,t}^F \right)^{1-\eta}, \tag{31}$$

where η denotes the bargaining power of workers employed formally. For simplification, we assume the same value for bargaining power in both sectors. Solving this problem we find that the surplus is shared according to following expressions

$$\mathcal{W}_t^F = x_t \mathcal{S}_t^F \tag{32}$$

$$\mathcal{V}_{n,t}^F = (1 - x_t)\mathcal{S}_t^F, \tag{33}$$

where $x_t = \frac{\eta}{\eta+\lambda_t^\omega(1-\eta)}$. Simplifying these expressions further and after solving for w_t^F we obtain the following wage equation in the formal sector

$$w_t^F = \eta\left(\frac{\Omega(\overline{\omega}_t)}{\lambda_t^\omega}a_t p_t^F + \frac{\lambda_t^\omega}{\lambda_{t+1}^\omega}\theta_t^F\varphi^F + \theta_t^I\varphi^I\right) + \eta(1-\delta^F)\frac{\lambda_t^\omega\varphi^F}{m_t^F}\left(\frac{1}{\lambda_t^\omega} - \frac{1}{\lambda_{t+1}^\omega}\right) + (1-\eta)\kappa. \tag{34}$$

The wage in the formal sector depends not only on terms that are determined in the debt contract but also on the labor market tightness in the informal sector. Equivalently, we define the total surplus, which an informal firm and a matched worker share as $\mathcal{S}_t^I = \mathcal{V}_{n,t}^I + \mathcal{W}_t^I$, where $\mathcal{W}_t^I = \frac{\mathcal{H}_{n,t}^I - \mathcal{H}_{u,t}}{\lambda_t^c}$. Using the expressions for the envelope condition in the informal sector and the net value of being employed in the informal sector, we further define the total surplus as

$$\begin{aligned} \mathcal{S}_t^I &= \gamma a_t p_t^I - \kappa \\ &\quad + (1-\delta^I)\beta\mathbb{E}_t\left\{\Lambda_{t,t+1}\left\{\mathcal{V}_{n,t+1}^I + \mathcal{W}_{t+1}^I\right\}\right\} \\ &\quad - \beta\mathbb{E}_t\left\{\Lambda_{t,t+1}\left\{\phi_t^I\mathcal{W}_{t+1}^I + \phi_t^F\mathcal{W}_{t+1}^F\right\}\right\}. \end{aligned} \tag{35}$$

Nash bargaining problem identical to (31) yields the following sharing conditions

$$\mathcal{W}_t^I = \eta\mathcal{S}_t^I \tag{36}$$

$$\mathcal{V}_{n,t}^I = (1-\eta)\mathcal{S}_t^I. \tag{37}$$

Performing the similar algebra on equations (35)-(37) we solve for the following wage equation in the informal sector

$$w_t^I = \eta\left(\gamma a_t p_t^I + \theta_t^I\varphi^I + \frac{\lambda_t^\omega}{\lambda_{t+1}^\omega}\theta_t^F\varphi^F\right) + (1-\eta)\kappa. \tag{38}$$

As in case of the formal sector wage, the wage of an informally employed worker is also affected by the labor market tightness in the formal sector.

3.7 Market clearing and equilibrium

In equilibrium, goods and factor markets clear and the resource constraints in both sectors are defined by

$$c_t^F + (\varphi^F/p_t^F)\upsilon_t^F = y_t^F(1-\mu_t G(\overline{\omega}_t)) \tag{39}$$

$$c_t^I + (\varphi^I/p_t^I)\upsilon_t^I = y_t^I, \tag{40}$$

where expression (39) states that the total output of formal firms net of the monitoring cost is used for the consumption of formal goods and creation of formal vacancies. Given the stochastic process for the evolution of aggregate productivity $\{a_t\}_{t=0}^{\infty}$, equilibrium in this economy is the sequence $\{y_t^F, y_t^I, c_t^F, c_t^I, c_t, n_t^F, n_t^I, u_t, v_t^F, v_t^I, r_t, p_t^F, p_t^I, w_t^F, w_t^I, \lambda_t^{\omega}, nw_t, \overline{\omega}_t\}_{t=0}^{\infty}$, which for given initial conditions satisfies equations (2), (3), (4), (5), (7), (8), (14) (15), (16), (20), (23), (27), (29), (34), (38), (39), (40), definitions of the job filling rate in the formal and informal sectors, job finding rates, labor market tightness as well as the expressions governing the evolution of the firm's and lender's share of revenues in the formal sector.[8]

4 Analysis of the model

4.1 Steady state equilibrium and calibration

In steady-state equilibrium, the aggregate productivity is constant and equal to $a = 1$. Given an empirically observed value of the bankruptcy rate, which is equivalent to $\Phi(\overline{\omega})$ in the model, we calibrate the threshold value of the idiosyncratic productivity shock, $\overline{\omega}$, and its standard deviation, σ_ω.[9] Specifically, we choose values of $\overline{\omega}$, σ_ω and μ, such that the value of $\Phi(\overline{\omega})$ equals the average quarterly share of non-performing loans in total loans in Brazil. We estimate this value to be about 3.24%.[10] At the same time, we target the formal firm's steady-state leverage ratio of 0.5, which is within the range of several estimates of this ratio for Brazilian firms. This calibration results in values $\sigma_\omega = 0.55$ and $\mu = 0.7$. We also target the steady-state unemployment rate of approximately 12% and the share of formally and informally employed workers of about 42% and 46%, respectively, as in Ulyssea (2010).

Multiplicative term γ in the production function of informal firms is set to 0.7, which corresponds to an estimated 30 percent productivity gap between formal and informal firms in Brazil (Perry et al., 2007). The cost of creating a formal vacancy is assumed to be significantly higher than the cost of creating an informal one. The elasticity of the matching function to unemployment and the bargaining power of workers equal $\alpha = \eta = 0.5$. The share of formal goods in the composite good, $\varkappa$, also equals 0.5. The elasticity of substitution between formal and informal goods, ρ, is chosen to be 0.3 (Ulyssea, 2010). The time preference parameter equals 0.9723, which corresponds to a real interest rate of about 11% on an annualized basis. Parameters of the autoregressive productivity process are chosen to be $\varrho = 0.97$ and $\sigma_a = 0.011$. The full list of variables with their values is shown in Table 2.

In steady state equilibrium flows into and out of unemployment are equal, $\delta^F n^F = \phi^F u$ and $\delta^I n^I = \phi^I u$. For given values of $\overline{\omega}$, σ_ω and μ the system of nonlinear equations characterizing

[8] The full set of the nonlinear equations is shown in Appendix B.

[9] Idiosyncratic productivity shock is assumed to be log-normally distributed.

[10] Information on non-performing loans in Brazil is available from the IMF. We use the average value of the "Non-performing loans to total gross loans" series available in the IMF Financial Soundness Indicators database for the corresponding period of 2005:Q1 - 2010:Q4.

Table 2: Parameters and their values

Parameter	Value	Notes
β	0.9723	Time preference parameter
$\Phi(\overline{\varpi})$	0.0342	Average quarterly share of non-performing loans to total loans
μ	0.7	Steady state monitoring cost
σ_ω	0.55	Standard deviation of the idiosyncratic productivity shock
γ	0.7	Term in the production function of informal firms
φ^F	0.24	Cost of opening a formal vacancy
φ^I	0.08	Cost of opening an informal vacancy
ψ^F	0.32	Efficiency of formal matching
ψ^I	0.48	Efficiency of informal matching
δ^F	0.05	Exogenous rate of separation of formal jobs
δ^I	0.08	Exogenous rate of separation of informal jobs
η	0.5	Bargaining power
α	0.5	Elasticity of the matching function
κ	0.26	Fixed unemployment benefit
$\varkappa$	0.5	Share of formal goods in the composite consumption good
ρ	0.3	Elasticity of substitution between formal and informal goods
ζ	0.85	Parameter of the evolution of net worth
ϱ	0.97	Persistence of the aggregate productivity process
σ_a	0.011	Standard deviation of the aggregate productivity shock

the equilibrium can be further simplified to the following system of nine equations in nine unknowns (θ^F, θ^I, c^F, c^I, c, n^F, n^I, p^F, p^I), which we numerically solve for the steady state equilibrium:

$$p^F = \frac{\lambda^\omega}{\Omega(\overline{\omega})}\left[\frac{\eta}{1-\eta}\left(\theta^F\varphi^F + \theta^I\varphi^I\right) + \kappa + \frac{(1-\beta(1-\delta^F))}{(1-\eta)\beta}\frac{\varphi^F}{m^F(\theta^F)}\right] \tag{41}$$

$$p^I = \frac{1}{\gamma}\left[\frac{\eta}{1-\eta}\left(\theta^F\varphi^F + \theta^I\varphi^I\right) + \kappa + \frac{(1-\beta(1-\delta^I))}{(1-\eta)\beta}\frac{\varphi^I}{m^I(\theta^I)}\right] \tag{42}$$

$$p^F = \varkappa(c^F)^{\rho-1}(c)^{1-\rho} \tag{43}$$

$$p^I = \varkappa(c^I)^{\rho-1}(c)^{1-\rho} \tag{44}$$

$$c = p^F c^F + p^I c^I \tag{45}$$

$$c^F = (1-\mu G(\overline{\omega}))n^F - (\varphi^F/p^F)\theta^F(1-n^F-n^I) \tag{46}$$

$$c^I = \gamma n^I - (\varphi^I/p^I)\theta^I(1-n^F-n^I) \tag{47}$$

$$\theta^F = \left(\frac{1}{\psi^F}\frac{\delta^F n^F}{1-n^F-n^I}\right)^{\frac{1}{1-\alpha}} \tag{48}$$

$$\theta^I = \left(\frac{1}{\psi^I}\frac{\delta^I n^I}{1-n^F-n^I}\right)^{\frac{1}{1-\alpha}}. \tag{49}$$

Equations (41) and (42) are obtained using the job creation condition and the wage equation, respectively for each sector. Equations (43) and (44) are derived from the consumption allocation equations, (46) and (47) are resource constraints, (48) and (49) are steady state values of labor market tightness in both sectors.

Selected parametrization results in the value of the steady state premium on external funds of 0.02. Steady state job finding rates for formal and informal employment are 0.17 and 0.30, respectively. Steady state wage in the formal sector is slightly higher than the wage in the informal sector. The ratio of the unemployment benefit to the formal wage, i.e. replacement ratio, equals 0.66.

A simulation exercise also shows how changes in the monitoring cost μ (which captures the extent of frictions in the financial sector) influence the steady state values of the key labor market variables. Results of this exercise are shown in Table 3. Changes in the monitoring cost result in sizable movements of workers between formal and informal employment in a steady state, though the effect on the unemployment rate is modest. An increase in the value of μ implies a higher cost of monitoring formal firms and a more severe moral hazard problem in the firm-lender relationship. Lower values of μ make borrowing cheaper allowing formal firms to expand further and hire more workers.

Table 3: Steady state values of labor market variables (in percent of total labor force) for different values of the monitoring cost μ.

μ	Formal employment	Informal employment	Unemployment
0	44.7	43.5	11.8
0.5	43.0	45.0	12.0
1	41.3	46.3	12.4

4.2 Model dynamics

We analyze the model's dynamic properties by log-linearizing the system of nonlinear equations above around the steady-state equilibrium.[11] In particular, we are interested in the effect of one standard deviation negative productivity shock on the evolution of key variables. In analyzing the model dynamics, we assume that one standard deviation negative productivity shock leads to a 25% increase in the monitoring cost. The elasticity of the monitoring cost to aggregate productivity shock is calibrated according to this assumption. Impulse responses of key model variables to the productivity shock are shown in Figure 3. Negative productivity shock leads to lower productivity in both types of firms; in addition, it reduces the net worth of firms that operate formally, which exacerbates credit frictions. Consequently, a higher level of frictions limits these firms' ability to create formal vacancies, which further limits workers' ability to enter formal employment. The share of workers employed formally declines and the share of workers employed informally increases. Due to matching frictions in the informal labor market and non-zero job creation costs, not all workers who are unable to enter formal employment find informal jobs and, thus, unemployment increases. The output of both sectors declines, but informal firms' output returns to its steady-state value relatively faster than the output of formal firms.

To assess the model's performance, we compute the relevant statistics based on the simulated data after applying the Hodrick-Prescott filter with the smoothing parameter $\lambda = 1600$. Correlations of the key variables with output and their relative volatilities are shown in Table 4. The statistics are calculated using the formal sector and total output. The latter is the sum of formal and informal sector output (in terms of the composite consumption good). Since the official GDP data commonly reflect the output of formally operating firms, model statistics calculated using both types of output are appropriate for comparison purposes.

The job finding rate in the formal sector, ϕ^F, is highly procyclical and about three times more volatile than the job finding rate in the informal sector. This result matches well with the empirical moments estimated using the Brazilian data. The model suggests that credit frictions that directly affect formal firms' ability to create new vacancies and maintain existing ones make access to formal jobs significantly more volatile than access to informal

[11] The full list of log-linearized equations is shown in Appendix C.

Table 4: Correlations with formal sector output, y^F, total output, y, and relative volatilities.

	ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.961	0.054	-0.573	0.464	-0.275
$\rho(x, y)$	0.961	-0.191	-0.358	0.231	-0.031
σ_x/σ_{y^F}	2.420	0.646	0.800	0.379	0.148
σ_x/σ_y	2.614	0.698	0.864	0.410	0.160

Notes: Superscripts F and I indicate formal and informal employment, respectively; ϕ denotes the job finding rate, u is the unemployment rate, n^F is the share of formal employment and n^I is the share of informal employment in total employment. Moments are calculated after detrending simulated series using a Hodrick-Prescott filter with the smoothing parameter $\lambda = 1600$.

jobs, as observed empirically. The job finding rate in the informal sector, ϕ^I, is generally acyclical. The share of workers employed formally is procyclical and more volatile than the share of informal workers. The unemployment rate is countercyclical but less volatile than output, which indicates that the model requires an additional amplification mechanism to increase the relative volatility of unemployment. Difficulty in replicating the empirically observed moments of the unemployment rate is a common shortcoming of simple real business cycle models with search and matching frictions in the labor market. Though the model performs well in replicating the differential behavior of job finding rates over the business cycle, overall, the volatility of job finding rates in both sectors and of the unemployment rate is lower than observed in the data. Nevertheless, qualitative predictions of the model are well in accordance with empirical observations on the dynamics of formal and informal employment over the business cycle.

4.3 Sensitivity analysis

We check the sensitivity of our results obtained using the baseline calibration to changes in key model parameters. Specifically, we look at implications of assuming different values for the steady state monitoring cost, μ, fixed unemployment benefits, κ, and standard deviation of the idiosyncratic productivity shock, σ_ω. Results of this sensitivity analysis are shown in Table 5.

Panels 5a and 5b show results of the simulation for two different values of the steady state monitoring cost μ. The lower value of μ represents an economy with a relatively lower level of financial frictions. In comparison, a higher value of μ indicates the existence of a severe moral hazard problem in the firm-lender relationship. The results highlight the importance of financial frictions for generating the differential behavior of job finding rates in both sectors. When we simulate the model with $\mu = 0.1$, shares of workers and job findings rates

Table 5: Correlations with formal sector output, y^F, total output, y, and relative volatilities.

(a) $\mu = 0.1$

	ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.993	0.831	-0.682	0.428	0.190
$\rho(x, y)$	0.998	0.725	-0.571	0.269	0.345
σ_x/σ_{y^F}	2.047	0.362	0.761	0.266	0.130
σ_x/σ_y	2.104	0.372	0.783	0.273	0.134

(b) $\mu = 0.9$

	ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.952	0.157	-0.582	0.479	-0.270
$\rho(x, y)$	0.956	-0.088	-0.370	0.250	-0.029
σ_x/σ_{y^F}	2.507	0.666	0.843	0.388	0.136
σ_x/σ_y	2.720	0.723	0.914	0.421	0.147

(c) $\kappa = 0.06$

	ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.939	-0.835	0.281	0.319	-0.438
$\rho(x, y)$	0.925	-0.944	0.485	0.058	-0.187
σ_x/σ_{y^F}	1.516	1.117	0.372	0.313	0.236
σ_x/σ_y	1.620	1.194	0.398	0.335	0.252

(d) $\kappa = 0.32$

	ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.964	0.964	-0.707	0.592	0.778
$\rho(x, y)$	0.988	0.894	-0.567	0.424	0.790
σ_x/σ_{y^F}	3.325	1.269	1.398	0.472	0.137
σ_x/σ_y	3.551	1.355	1.493	0.504	0.146

(e) $\sigma_\omega = 0.15$

	ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.659	-0.051	-0.406	0.867	-0.944
$\rho(x, y)$	0.839	-0.556	-0.001	0.483	-0.668
σ_x/σ_{y^F}	4.191	3.344	1.260	0.828	0.536
σ_x/σ_y	8.660	6.909	2.604	1.711	1.109

(f) $\sigma_\omega = 0.75$

	ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.926	0.976	-0.637	0.285	0.726
$\rho(x, y)$	0.921	0.964	-0.598	0.203	0.758
σ_x/σ_{y^F}	1.665	0.943	0.891	0.181	0.143
σ_x/σ_y	1.617	0.916	0.865	0.175	0.139

Notes: μ is the steady state monitoring costs, κ denotes the fixed unemployment benefit and σ_ω is the standard deviation of the idiosyncratic productivity shock. Superscripts F and I indicate formal and informal employment, respectively; ϕ denotes the job finding rate, u is the unemployment rate, n^F is the share of formal employment and n^I is the share of informal employment in total employment. Moments are calculated after detrending simulated series using a Hodrick-Prescott filter with the smoothing parameter $\lambda = 1600$.

in both sectors become highly procyclical, which contrasts our empirical observations using the Brazilian data. On the contrary, assuming a higher level of the monitoring cost (and, thus, a higher degree of credit frictions) generates the differential behavior of job finding rates and worker shares as observed in the data.

It is known in the literature that the value of fixed unemployment benefits strongly influences the cyclical behavior of real business cycle models with search and matching frictions in the labor market. In this regard, we simulate the model with two different values of the fixed unemployment benefits parameter κ. We choose the values of κ such that we simulate the following two scenarios. In the first scenario, Table 5c, we choose the value of κ such that the replacement ratio (ratio of the unemployment benefit to the steady state wage in the formal sector) is approximately equal to 15%. In equilibrium, this results in a lower level of steady state unemployment and, consequently, its lower sensitivity to aggregate productivity shocks. As expected, the relative volatility of key variables declines but the cyclical behavior of some variables changes. The job finding rate in the informal sector becomes highly countercyclical and, contrary to intuition and empirical observations, the unemployment rate becomes somewhat procyclical. In the second scenario, Table 5d, we choose the value of κ, such that the replacement ratio equals 80%. Under this calibration, the unemployment rate is highly countercyclical, as observed in the data. Compared to our results with baseline calibration, assuming a relatively higher value of unemployment benefits induces job finding rates and worker shares in both sectors to be procyclical.

We also simulate the model assuming two different values for the standard deviation of the idiosyncratic productivity shock, σ_ω. Given the expected value of the idiosyncratic productivity shock, changes in σ_ω lead to different steady state values of the threshold value of the idiosyncratic productivity shock, $\overline{\omega}$. In particular, smaller values of σ_ω shift the mode of the log-normal distribution to the right and the latter approaches the normal distribution. This leads to a higher value of the productivity shock threshold, $\overline{\omega}$ and the impact of financial frictions strengthens. The opposite is true when we increase the value of σ_ω. Under this calibration, the positive skewness of the log-normal distribution increases and the threshold level of the idiosyncratic productivity shock, $\overline{\omega}$, declines. The latter implies that the impact of financial frictions on formal firms declines. This is reflected in the results of simulations shown in Tables 5e - 5f. Under a high level of σ_ω and the resulting weak effect of financial frictions, the model behaves in a manner very similar to a case, when we assume a low monitoring cost of $\mu = 0.1$. In this case, frictions in the financial market are not pronounced and, thus, job finding rates and shares of workers in both sectors are procyclical. Lowering the value of σ_ω and, thus, strengthening the effect of financial frictions on formal firms results in the differential behavior of job finding rates and shares of workers across sectors. The volatility of key labor market variables also increases.

Overall, simulations suggest that the model requires simultaneously (i) a relatively higher level of unemployment benefits (in the formal sector) to generate the countercyclical unemployment rate; and (ii) a relatively higher degree of financial frictions to generate the differential behavior of job finding rates and labor shares between the two sectors.

4.4 Role of wage rigidity in the formal sector

Under baseline parametrization, the relative volatility of formal wage to the wage in the informal sector, $\sigma_{w^F}/\sigma_{w^I}$, equals 2.22. This seems implausible since it is unlikely that the wage is a primary adjustment tool over the business cycle in formal firms in emerging economies, where contractual restrictions and other labor market regulations would prevent firms from readily adjusting on this margin. To test how the model performs under wage rigidity, we exogenously introduce some degree of wage rigidity in the formal sector. In particular, we assume that the period t wage in the formal sector is determined according to the following equation:

$$w_t^F = z w_{N,t}^F + (1 - z) w_{t-1}^F, \tag{50}$$

Table 6: Correlations with formal sector output, y^F, total output, y, and relative volatilities, model with exogenous wage rigidity.

	ϕ^F	ϕ^I	u	n^F	n^I
$\rho(x, y^F)$	0.748	0.306	-0.691	0.816	-0.866
$\rho(x, y)$	0.848	0.120	-0.558	0.691	-0.791
σ_x/σ_{y^F}	3.499	1.938	1.172	0.462	0.154
σ_x/σ_y	4.541	2.515	1.521	0.599	0.200

Notes: Superscripts F and I indicate formal and informal employment, respectively; ϕ denotes the job finding rate, u is the unemployment rate, n^F is the share of formal employment and n^I is the share of informal employment in total employment. Moments are calculated after detrending simulated series using a Hodrick-Prescott filter with the smoothing parameter $\lambda = 1600$.

where $w^F_{N,t}$ equals equation (34), the wage in the formal sector determined through the Nash bargaining mechanism. In simulations with this additional wage equation, we choose $z = 0.3$. Other parameter values remain the same.

Impulse responses of the variables to a standard deviation negative productivity shock are shown in Figure 4. The qualitative behavior of key endogenous variables in the augmented model is very similar to the model's behavior with no wage rigidity, as shown in Table 6. Correlations of labor market variables with output for both sectors do not considerably differ from previous baseline results. The significant difference is in terms of relative volatilities. The volatility of the job finding rate in the formal sector increases and it approaches the value observed in the data. The relative volatility of the unemployment rate increases too but still falls short of the empirically observed value. So, while performing well in other dimensions, the model still appears to require an additional amplification mechanism to increase the relative volatility of the unemployment rate.[12]

5 Conclusions

We build and analyze a dynamic stochastic general equilibrium model, in which a negative aggregate productivity shock leads to a substantial movement of the labor force from the formal to the informal sector over the business cycle. In the model, changes in financial conditions over the business cycle directly affect the formal firms' ability to borrow and hinder the creation of new jobs when aggregate productivity declines. We show that varying over the business cycle access to credit for formal firms leads to strongly procyclical and volatile labor market tightness and job-finding rate in the formal sector, as observed empirically. This, subsequently, leads to countercyclical unemployment and the countercyclical share of

[12] The introduction of an endogenous job destruction might be one of such mechanisms, though this is beyond the immediate scope of this paper.

informal workers in the economy.

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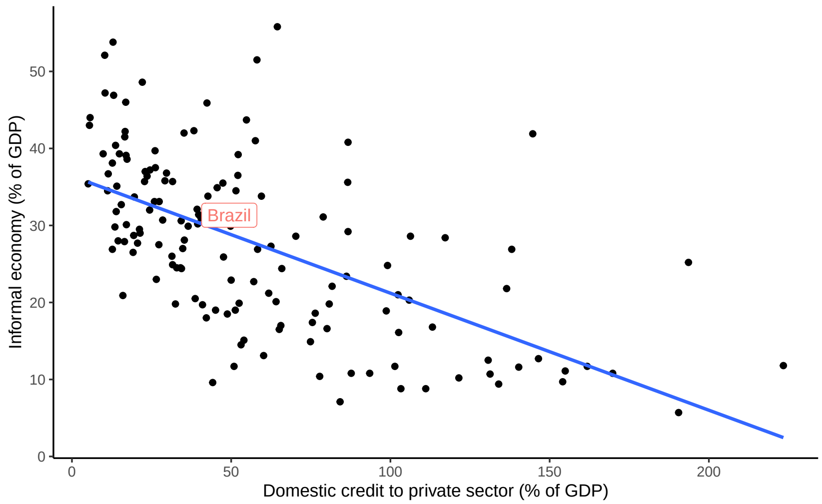
A Tables and figures

Figure 1: Informality and private credit.



Note: Observations for 2017 are plotted.
Source: The World Bank, Medina and Schneider (2019).

Table 7: Annual transition rates (in %).

	Formally employed	Informally employed	Unemployed
Formally employed	89.5	6.0	4.5
Informally employed	32.2	57.7	10.1
Unemployed	30.9	22.2	46.9

Notes: PME data for 2002 - 2007 is used. Transition rates are computed using information from the forth and eighth interviews of an individual.

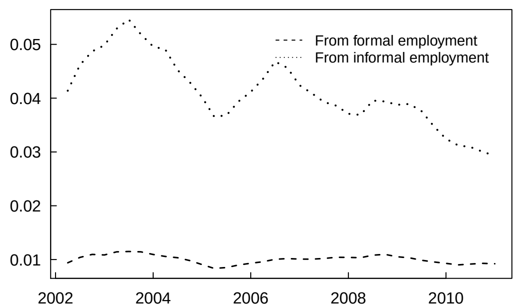
Table 8: Correlation and relative volatility of labor market variables in Brazil (extended definition of informal employment).

x	ϕ^F	ϕ^I	λ^F	λ^I	τ^{FI}	τ^{IF}	u	n^F
$\rho(y,x)$	0.50	-0.22	-0.16	-0.37	0.10	0.31	-0.50	0.28
σ_x/σ_y	4.38	3.39	4.33	5.69	2.08	2.54	2.87	0.58

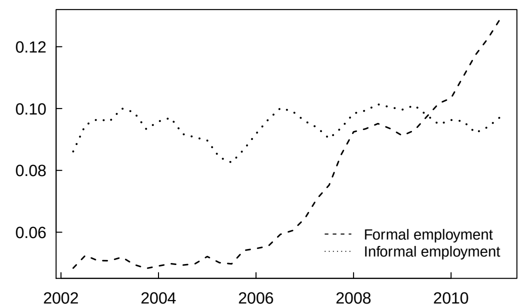
Notes: Superscripts F and I indicate formal and informal employment, respectively; y refers to real quarterly GDP, x refers to the other variables: ϕ denotes the job finding rate, λ is the job separation rate, τ is the direct transition rate between formal and informal employment, u is the unemployment rate and n^F is the share of formal employment in total employment. Statistics are computed after taking natural logarithm and detrending original series using the HP filter with the smoothing parameter $\lambda = 1600$. Informal employment includes both informal salaried employment and informal self-employment.

Figure 2: Evolution of labor market variables in Brazil.

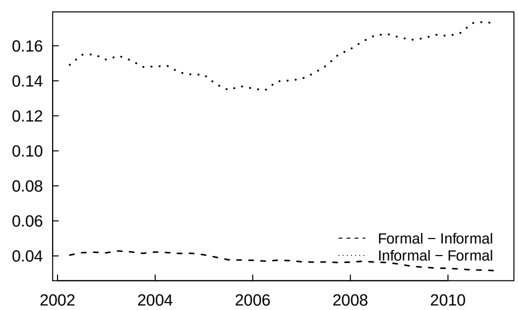
(a) Job separation rates



(b) Job finding rates



(c) Direct job to job transition rates



Notes: Plotted series are quarterly averages of originally calculated monthly series. All series are smoothed using a moving average filter.

Figure 3: Impulse responses to one s.d. negative shock to aggregate productivity.

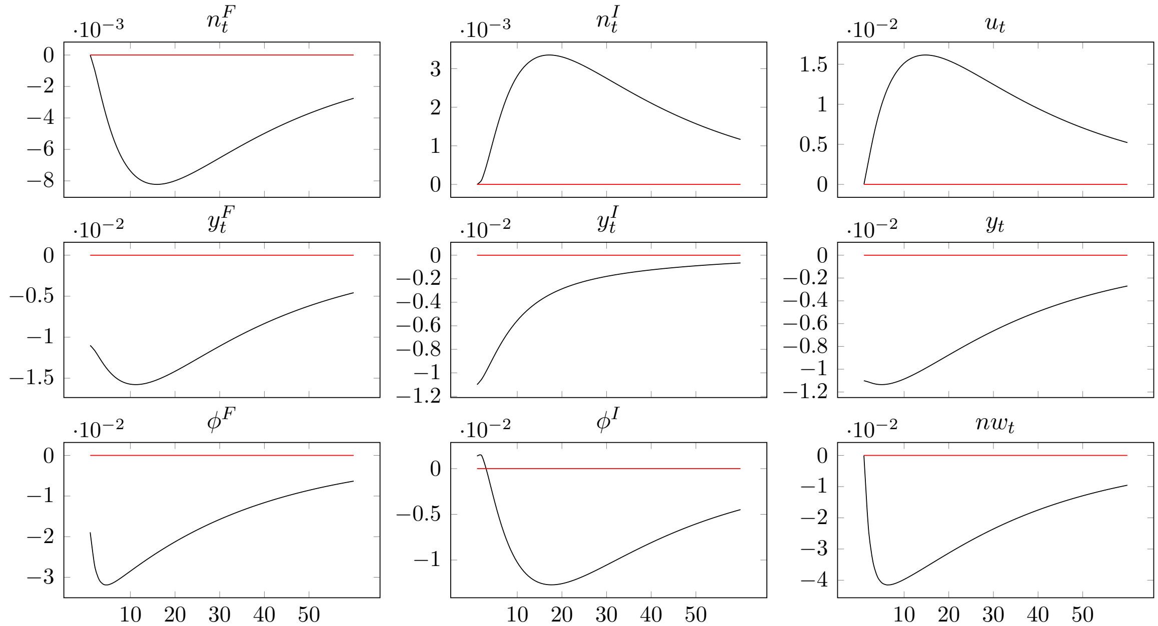
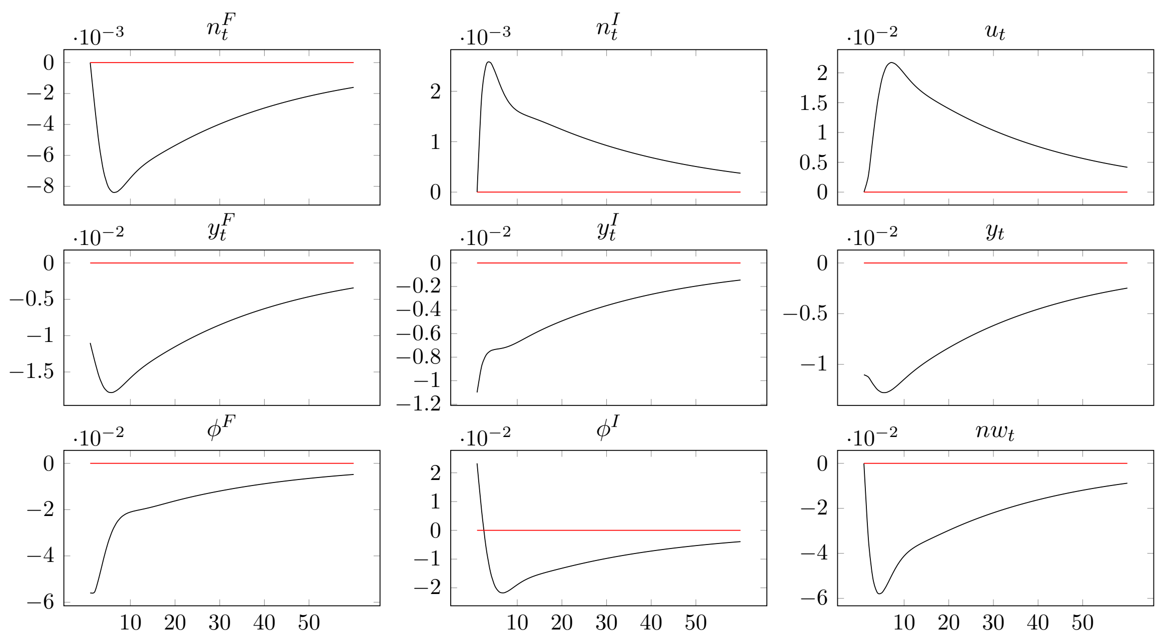


Figure 4: Impulse responses to one s.d. negative shock to aggregate productivity, model with exogenous wage rigidity.



B Nonlinear equations

$$\frac{\lambda_t^{\omega}\varphi^F}{m_t^F} = \beta\mathbb{E}_t\left\{\Lambda_{t,t+1}\left\{\Omega(\overline{\omega}_{t+1})a_{t+1}p_{t+1}^F - \lambda_{t+1}^{\omega}w_{t+1}^F + (1-\delta^F)\frac{\lambda_{t+1}^{\omega}\varphi^F}{m_{t+1}^F}\right\}\right\} \tag{B.1}$$

$$\frac{\varphi^I}{m_t^I} = \beta\mathbb{E}_t\left\{\Lambda_{t,t+1}\left\{\gamma a_{t+1}p_{t+1}^I - w_{t+1}^I + (1-\delta^I)\frac{\varphi^I}{m_{t+1}^I}\right\}\right\} \tag{B.2}$$

$$\begin{aligned} w_t^F &= \eta\left(\frac{\Omega(\overline{\omega}_t)}{\lambda_t^{\omega}}a_t p_t^F + \frac{\lambda_t^{\omega}}{\lambda_{t+1}^{\omega}}\theta_t^F\varphi^F + \theta_t^I\varphi^I\right)\dots \\ &\quad + \eta(1-\delta^F)\frac{\lambda_t^{\omega}\varphi^F}{m_t^F}\left(\frac{1}{\lambda_t^{\omega}} - \frac{1}{\lambda_{t+1}^{\omega}}\right) + (1-\eta)\kappa \end{aligned} \tag{B.3}$$

$$w_t^I = \eta\left(\gamma a_t p_t^I + \theta_t^I\varphi^I + \frac{\lambda_t^{\omega}}{\lambda_{t+1}^{\omega}}\theta_t^F\varphi^F\right) + (1-\eta)\kappa \tag{B.4}$$

$$c_t^{-\frac{1}{\xi}} = \lambda_t^c \tag{B.5}$$

$$p_t^F = \varkappa(c_t^F)^{\rho-1}(c_t)^{1-\rho} \tag{B.6}$$

$$p_t^I = (1-\varkappa)(c_t^I)^{\rho-1}(c_t)^{1-\rho} \tag{B.7}$$

$$c_t = p_t^F c_t^F + p_t^I c_t^I \tag{B.8}$$

$$\lambda_t^c = \beta(1+r_t)\mathbb{E}_t\lambda_{t+1}^c \tag{B.9}$$

$$y_t^F = \frac{1}{(1-\mu_t G(\overline{\omega}_t))}\left(c_t^F + (\varphi^F/p_t^F)\upsilon_t^F\right) \tag{B.10}$$

$$y_t^F = a_t n_t^F \tag{B.11}$$

$$y_t^I = c_t^I + (\varphi^I/p_t^I)\upsilon_t^I \tag{B.12}$$

$$y_t^I = \gamma a_t n_t^I \tag{B.13}$$

$$n_{t+1}^F = (1-\delta^F)n_t^F + m_t^F\upsilon_t^F \tag{B.14}$$

$$n_{t+1}^I = (1-\delta^I)n_t^I + m_t^I\upsilon_t^I \tag{B.15}$$

$$u_{t+1} = (1-\phi_t^F - \phi_t^I)u_t + \delta^F n_t^F + \delta^I n_t^I \tag{B.16}$$

$$nw_{t+1} = \zeta\left(a_t p_t^F n_t^F - (\mathcal{C}_t^F - nw_t)\left(1 + \frac{\mu_t G(\overline{\omega}_t)a_t p_t^F n_t^F}{\mathcal{C}_t^F - nw_t}\right)\right) \tag{B.17}$$

$$\theta_t^F = \frac{\upsilon_t^F}{u_t} \tag{B.18}$$

$$\theta_t^I = \frac{\upsilon_t^I}{u_t} \tag{B.19}$$

$$m_t^F = \psi^F(\theta_t^F)^{-\alpha} \tag{B.20}$$

$$m_t^I = \psi^I(\theta_t^I)^{-\alpha} \tag{B.21}$$

$$\phi_t^F = \psi^F(\theta_t^F)^{1-\alpha} \tag{B.22}$$

$$\phi_t^I = \psi^I(\theta_t^I)^{1-\alpha} \tag{B.23}$$

$$\lambda_t^\omega = \frac{\Gamma'(\overline{\omega}_t)}{\Gamma'(\overline{\omega}_t) - \mu_t G'(\overline{\omega}_t)} \tag{B.24}$$

$$w_t^F n_t^F + \varphi^F \upsilon_t^F - nw_t = (\Gamma(\overline{\omega}_t) - \mu_t G(\overline{\omega}_t))\, a_t p_t^F n_t^F \tag{B.25}$$

$$\Gamma(\overline{\omega}_t) = \int_0^{\overline{\omega}_t} \omega d\Phi(\omega) + \int_{\overline{\omega}_t}^{\infty} \overline{\omega}_t d\Phi(\omega) = \overline{\omega}_t(1 - \Phi(\overline{\omega}_t)) + G(\overline{\omega}_t) \tag{B.26}$$

$$G(\overline{\omega}_t) = \int_0^{\overline{\omega}_t} \omega d\Phi(\omega) \tag{B.27}$$

$$\Gamma'(\overline{\omega}_t) = 1 - \Phi(\overline{\omega}_t) \tag{B.28}$$

$$G'(\overline{\omega}_t) = \overline{\omega}_t \Phi'(\overline{\omega}_t) \tag{B.29}$$

$$\Omega(\overline{\omega}_t) = (1 - \Gamma(\overline{\omega}_t)) + \lambda_t^\omega (\Gamma(\overline{\omega}_t) - \mu_t G(\overline{\omega}_t)) \tag{B.30}$$

$$\mu_t = h(a_t) \tag{B.31}$$

C Log-linear equations

Log-linearized variables are denoted by "hat" and variables without time subscripts are their steady state values. The following system of 21 equations defines the evolution of the following 21 variables $\widehat{p}_t^F$, $\widehat{p}_t^I$, $\widehat{c}_t^F$, $\widehat{c}_t^I$, $\widehat{c}_t$, $\widehat{y}_t^F$, $\widehat{y}_t^I$, $\widehat{a}_t$, $\widehat{n}_t^F$, $\widehat{n}_t^I$, $\widehat{\upsilon}_t^F$, $\widehat{\upsilon}_t^I$, $\widehat{u}_t$, $\widehat{r}_t$, $\widehat{w}_t^F$, $\widehat{w}_t^I$, $\widehat{\theta}_t^F$, $\widehat{\theta}_t^I$, $\widehat{\lambda}_t^\omega$, $\widehat{nw}_t$, $\widehat{\overline{\omega}}_t$.

$$\begin{aligned}\frac{\lambda^\omega \varphi^F(1+r)}{\psi^F(\theta^F)^{-\alpha}}(\widehat{\lambda}_t^\omega + \alpha\widehat{\theta}_t^F) = &- \frac{r}{1+r}\left[\Omega(\overline{\omega})p^F - \lambda^\omega w^F + (1-\delta^F)\frac{\lambda^\omega\varphi^F}{\psi^F(\theta^F)^{-\alpha}}\right]\widehat{r}_t \\ &- \Omega'(\overline{\omega})\overline{\omega}p^F\widehat{\overline{\omega}}_{t+1} + \Omega(\overline{\omega})p^F(\widehat{p}_{t+1}^F + \widehat{a}_{t+1}) - \lambda^\omega w^F(\widehat{w}_{t+1}^F + \widehat{\lambda}_t^\omega) \\ &+ \lambda^\omega p^F(\Gamma(\overline{\omega}) - \mu G(\overline{\omega}))\widehat{\lambda}_{t+1}^\omega - \lambda^\omega p^F h'(a)G(\overline{\omega})\widehat{a}_{t+1} \\ &+ (1-\delta^F)\frac{\lambda^\omega\varphi^F}{\psi^F(\theta^F)^{-\alpha}}(\widehat{\lambda}_{t+1}^\omega + \alpha\widehat{\theta}_{t+1}^F)\end{aligned} \tag{C.1}$$

$$\begin{aligned}\frac{\varphi^I(1+r)}{\psi^I(\theta^I)^{-\alpha}}\alpha\widehat{\theta}_t^I = &- \frac{r}{1+r}\left[\gamma p^I - w^I + (1-\delta^I)\frac{\varphi^I}{\psi^I(\theta^I)^{-\alpha}}\right]\widehat{r}_t + \gamma p^I(\widehat{p}_{t+1}^I + \widehat{a}_{t+1}) \\ &- w^I\widehat{w}_{t+1}^I + (1-\delta^I)\frac{\varphi^I}{\psi^I(\theta^I)^{-\alpha}}\alpha\widehat{\theta}_{t+1}^I\end{aligned} \tag{C.2}$$

$$\begin{aligned}\frac{w^F}{\eta}\widehat{w}_t^F = &\frac{\Omega(\overline{\omega})}{\lambda^\omega}(\widehat{p}_t^F + \widehat{a}_t) - \frac{p^F}{\lambda^\omega}(1-\Gamma(\overline{\omega}))\widehat{\lambda}_t^\omega + \frac{\Omega'(\overline{\omega})}{\lambda^\omega}p^F\overline{\omega}\,\widehat{\overline{\omega}}_t \\ &- h'(a)G(\overline{\omega})p^F\widehat{a}_t + \theta^F\varphi^F(\widehat{\theta}_t^F + \widehat{\lambda}_t^\omega - \widehat{\lambda}_{t+1}^\omega) + \theta^I\varphi^I\widehat{\theta}_t^I\end{aligned}$$

$$+ (1-\delta^F)\frac{\varphi^F}{\psi^F(\theta^F)^{-\alpha}}(\widehat{\lambda}^\omega_{t+1} - \widehat{\lambda}^\omega_t) \tag{C.3}$$

$$\frac{w^I}{\eta}\widehat{w}^I_t = \gamma p^I(\widehat{p}^I_t + \widehat{a}_t) + \theta^F\varphi^F(\widehat{\theta}^F_t + \widehat{\lambda}^\omega_t - \widehat{\lambda}^\omega_{t+1}) + \theta^I\varphi^I\widehat{\theta}^I_t \tag{C.4}$$

$$\widehat{c}_{t+1} = \widehat{c}_t + \xi\frac{r}{1+r}\widehat{r}_t \tag{C.5}$$

$$\widehat{p}^F_t = (\rho-1)\widehat{c}^F_t + (1-\rho)\widehat{c}_t \tag{C.6}$$

$$\widehat{p}^I_t = (\rho-1)\widehat{c}^I_t + (1-\rho)\widehat{c}_t \tag{C.7}$$

$$\widehat{c}_t = \frac{p^Fc^F}{c}(\widehat{p}^F_t + \widehat{c}^F_t) + \frac{p^Ic^I}{c}(\widehat{p}^I_t + c^I_t) \tag{C.8}$$

$$(1-\mu G(\overline{\omega}))(\widehat{y}^F_t + \widehat{p}^F_t) = \frac{c^F}{y^F}(\widehat{c}^F_t + \widehat{p}^F_t) + \frac{\varphi^F\upsilon^F}{p^Fy^F}\widehat{\upsilon}^F_t + h'(a)G(\overline{\omega})\widehat{a}_t + \mu G'(\overline{\omega})\overline{\omega}\;\widehat{\overline{\omega}}_t \tag{C.9}$$

$$\widehat{y}^F_t = \widehat{a}_t + \widehat{n}^F_t \tag{C.10}$$

$$\widehat{y}^I_t + \widehat{p}^I_t = \frac{c^F}{y^I}(\widehat{c}^I_t + \widehat{p}^I_t) + \frac{\varphi^I\upsilon^I}{p^Iy^I}\widehat{\upsilon}^I \tag{C.11}$$

$$\widehat{y}^I_t = \widehat{a}_t + \widehat{n}^I_t \tag{C.12}$$

$$\widehat{n}^F_{t+1} = (1-\delta^F)\widehat{n}^F_t + \frac{\psi^Fu^\alpha(\upsilon^F)^{1-\alpha}}{n^F}(\alpha\widehat{u}_t + (1-\alpha)\widehat{\upsilon}^F_t) \tag{C.13}$$

$$\widehat{n}^I_{t+1} = (1-\delta^I)\widehat{n}^I_t + \frac{\psi^Iu^\alpha(\upsilon^I)^{1-\alpha}}{n^I}(\alpha\widehat{u}_t + (1-\alpha)\widehat{\upsilon}^I_t) \tag{C.14}$$

$$u\widehat{u}_t = -n^F\widehat{n}^F_t - n^I\widehat{n}^I_t \tag{C.15}$$

$$\widehat{\theta}^F_t = \widehat{\upsilon}^F_t - \widehat{u}_t \tag{C.16}$$

$$\widehat{\theta}^I_t = \widehat{\upsilon}^I_t - \widehat{u}_t \tag{C.17}$$

$$\begin{aligned} nw\widehat{nw}_{t+1} = \zeta\Big(& p^Fn^F\left((1-\mu G(\overline{\omega}))(\widehat{p}^F_t + \widehat{a}_t + \widehat{n}^F_t) - \mu G'(\overline{\omega})\overline{\omega}\;\widehat{\overline{\omega}}_t - h'(a)G'(\overline{\omega})\widehat{a}_t\right) \\ & - w^Fn^F(\widehat{w}^F_t + \widehat{n}^F_t) - \varphi^F\upsilon^F\widehat{\upsilon}^F_t + nw\widehat{nw}_t\Big) \end{aligned} \tag{C.18}$$

$$(\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega}))\,\widehat{\lambda}^\omega_t = h'(a)G'(\overline{\omega})\widehat{a}_t - \left(\Gamma''(\overline{\omega}) - \mu G''(\overline{\omega}) - \frac{\Gamma''(\overline{\omega})}{\lambda^\omega}\right)\overline{\omega}\;\widehat{\overline{\omega}}_t \tag{C.19}$$

$$\begin{aligned} \widehat{p}^F + \widehat{a}_t + \widehat{n}^F_t = & \frac{1}{\Gamma(\overline{\omega}) - \mu G(\overline{\omega})}\left(\frac{w^F}{p^F}(\widehat{w}^F_t + \widehat{n}^F_t) + \frac{\varphi^F\upsilon^F}{p^Fn^F}\widehat{\upsilon}^F_t - \frac{nw}{p^Fn^F}\widehat{nw}_t\right) \\ & - \frac{\Gamma'(\overline{\omega}) - \mu G'(\overline{\omega})}{\Gamma(\overline{\omega}) - \mu G(\overline{\omega})}\overline{\omega}\;\widehat{\overline{\omega}}_t + \frac{h'(a)G(\overline{\omega})}{\Gamma(\overline{\omega}) - \mu G(\overline{\omega})}\widehat{a}_t \end{aligned} \tag{C.20}$$

$$\widehat{a}_t = \varrho\widehat{a}_{t-1} + \varepsilon_t \tag{C.21}$$